Ratio test will not be helpful here. Look for a^n, !, ...

Quiz 8 11.6: Absolute/Conditional Convergence, Ratio Test, and Root Test

- Determine the convergence/divergence of the following series.
- If the series converges, classify convergence as absolute or conditional.
    - Remember that to conclude a series is conditionally convergent, you need to show both that it is convergent, but not absolutely convergent
- Show all work neatly and with extremetly clear presentation, naming any test used and showing that all requirements have been met.

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1) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^4+1}}$

Helpful to tell readers what you are planning to do

check for absolute convergence

$\sum |a_n| = \sum \frac{1}{\sqrt{n^4+1}}$. Compare to $\sum \frac{1}{n^2}$, which converges ($p=2>1$)

Since $0 < \frac{1}{\sqrt{n^4+1}} < \frac{1}{\sqrt{n^4}} = \frac{1}{n^2}$, by comparison the series $\sum \frac{1}{\sqrt{n^4+1}}$ converges

extra statement makes it more clear

So $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^4+1}}$ is absolutely convergent

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2) $\sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{n}$

Watch the parenthesis $(2n)!$, $2n!$ are not the same

Applying the ratio test

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{(-1)^{n+1}(2(n+1))!}{n+1} \cdot \frac{n}{(-1)^n (2n)!} \right|$$

$$= \frac{(2n+2)!}{(2n)!} \frac{n}{n+1} = \frac{(2n+2)(2n+1)\cancel{(2n)!}\, n}{\cancel{(2n)!}\,(n+1)}$$

$$= \frac{2\cancel{(n+1)}(2n+1)n}{\cancel{n+1}} = 4n^2+2n \quad L = \lim_{n\to\infty} (4n^2+2n) = \infty$$

so given series diverges

3) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n^n}$ All powers of n suggests Root test

$$\sqrt[n]{|a_n|} = \sqrt[n]{\left|\frac{(-1)^n 2^n}{n^n}\right|} = \frac{2}{n}$$

$\lim_{n\to\infty} \frac{2}{n} = 0$ so given series is Abs. Conv. by the root test.

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4) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

Check for Abs Convergence

Consider $\sum_{n=1}^{\infty} \left|\frac{(-1)^n \ln n}{n}\right| = \sum \frac{\ln n}{n}$. compare to $\sum \frac{1}{n}$ (diverges)

$\ln n > 1$ for $n > e$ so $\frac{\ln n}{n} > \frac{1}{n} > 0$

so by comparison $\sum \frac{\ln n}{n}$ diverges which means $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$ is not absolutely conv, but still might converge

Apply AST since $\sum \frac{(-1)^n \ln n}{n}$ is alternating $b_n = \frac{\ln n}{n}$

i) decreasing. Cannot say clearly $b_{n+1} < b_n$ so use

$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2} < 0$ when $1 - \ln x < 0$

$1 < \ln x$

$x > e$

ii) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{\ln n}{n} \overset{L}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$

$\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$ converges by AST

Since $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$ converges but is not absolutely convergent it is conditionally convergent